Aufgabe 1

a) Berechne analytisch die Ausgangsspannung $u_a(t)$

Formel Addierverstärker:

$$u_a(t) = -\left(\frac{R_f}{R_1}u_{e1}(t) + \frac{R_f}{R_2}u_{e2}(t)\right)$$

Daraus ergibt sich:

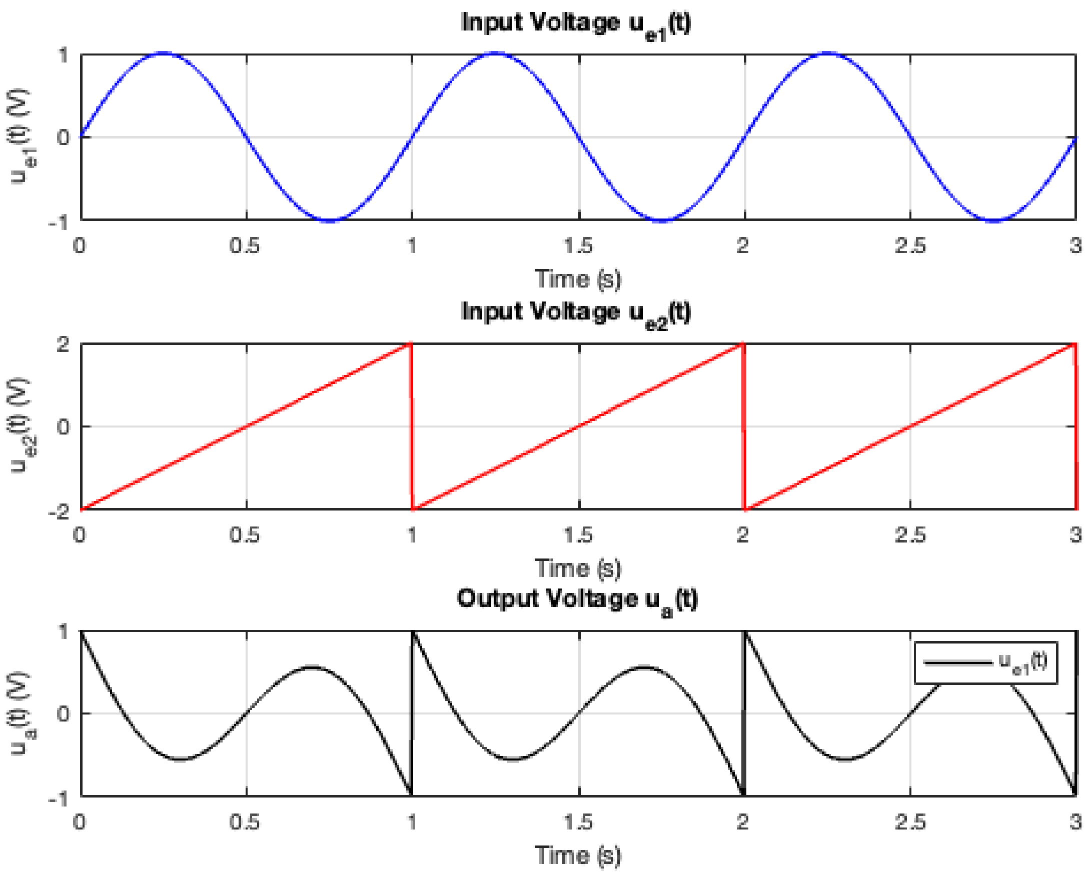
$$u_a(t) = -\left(\frac{R_3}{R_1}u_{e1}(t) + \frac{R_3}{R_2}u_{e2}(t)\right)$$

Einsetzen:

$$u_a(t) = -\left(\frac{20\,\mathrm{k\Omega}}{20\,\mathrm{k\Omega}}U_1 \cdot \sin(t) + \frac{20\,\mathrm{k\Omega}}{40\,\mathrm{k\Omega}}u_2(t)\right)$$

$$u_a(t) = -\left(U_1 \cdot \sin(t) + 0.5 \cdot u_{e2}(t)\right)$$

b) Zeichne den Kurvenverlauf von $u_a(t)$



Aufgabe 2

a) die Fläche unter der Funktion im Intervall $[0, 6]$.

Step 1: Calculate m_0 (Total Area Under the Curve)
Break the integral into two intervals:

$$m_0 = \int_0^2 s(t)\,dt + \int_2^6 s(t)\,dt$$

Interval $[0, 2]$

$$\int_0^2 (-3e^{-t} + 3)\,dt = -3\int_0^2 e^{-t}\,dt + 3\int_0^2 dt$$

Compute each term:
First term:

$$-3\int_0^2 e^{-t}\,dt = -3\left(-e^{-t}\right)\Big|_0^2 = -3\left(-e^{-2} + 1\right) = -3(1 - e^{-2})$$

Second term:

$$3\int_0^2 dt = 3t\Big|_0^2 = 3(2 - 0) = 6$$

Combine terms:

$$m_{0_1} = -3(1 - e^{-2}) + 6 = 3e^{-2} + 3$$

Interval $[2, 6]$

$$\int_2^6 \left(-3e^{-t} + 3e^{-(t-2)}\right)dt = -3\int_2^6 e^{-t}dt + 3\int_2^6 e^{-(t-2)}dt$$

First term:

$$-3\int_2^6 e^{-t}dt = -3\left(-e^{-t}\right)\Big|_2^6 = -3\left(-e^{-6} + e^{-2}\right) = -3(e^{-2} - e^{-6})$$

Second term (using substitution $u = t - 2$):

$$3\int_2^6 e^{-(t-2)}dt = 3\int_0^4 e^{-u}du = 3\left(-e^{-u}\right)\Big|_0^4 = 3(1 - e^{-4})$$

Combine terms:

$$m_{0_2} = -3(e^{-2} - e^{-6}) + 3(1 - e^{-4}) = -3e^{-2} + 3e^{-6} + 3 - 3e^{-4}$$

Total m_0

$$\begin{aligned} m_0 &= m_{0_1} + m_{0_2} \\ &= \left(3e^{-2} + 3\right) + \left(-3e^{-2} + 3e^{-6} + 3 - 3e^{-4}\right) \\ &= (3e^{-2} - 3e^{-2}) + (3 + 3) + (3e^{-6} - 3e^{-4}) \\ &= 6 + 3e^{-6} - 3e^{-4} \end{aligned}$$

Numerical value:

$$m_0 \approx 6 + 3(0.00247875) - 3(0.0183156) = 6 + 0.00743625 - 0.0549468 \approx 5.9525$$

b) die Verzögerungszeit $t_d = \frac{m_1}{m_0}$ mittels Impulsschwerpunkt.

Calculate m_1 (First Moment of the Signal)
Break the integral into two intervals:

$$m_1 = \int_0^2 t \cdot s(t)\, dt + \int_2^6 t \cdot s(t)\, dt$$

Interval $[0, 2]$:

$$\int_0^2 t(-3e^{-t} + 3)dt = -3\int_0^2 te^{-t}dt + 3\int_0^2 tdt$$

First term (integration by parts):
Let $u = t$, $dv = e^{-t}dt$. Then $du = dt$, $v = -e^{-t}$.

$$\int te^{-t}dt = -te^{-t} - e^{-t} + C$$

Evaluate:

$$-3\int_0^2 te^{-t}dt = -3\left(-te^{-t} - e^{-t}\right)\Big|_0^2 = -3\left(\left(-2e^{-2} - e^{-2}\right) - (0 - 1)\right) = -3\left(-3e^{-2} + 1\right) = 9e^{-2} - 3$$

Second term:

$$3\int_0^2 tdt = 3\left(\frac{t^2}{2}\right)\Big|_0^2 = 3\left(\frac{4}{2} - 0\right) = 6$$

Combine terms:

$$m_{1_1} = (9e^{-2} - 3) + 6 = 9e^{-2} + 3$$

Interval $[2, 6]$:

$$\int_2^6 t\left(-3e^{-t} + 3e^{-(t-2)}\right)dt = -3\int_2^6 te^{-t}dt + 3\int_2^6 te^{-(t-2)}dt$$

First term (integration by parts):

$$\int te^{-t}dt = -te^{-t} - e^{-t} + C$$

Evaluate:

$$-3\int_2^6 te^{-t}dt = -3\left(-te^{-t} - e^{-t}\right)\Big|_2^6 = -3\left(\left(-6e^{-6} - e^{-6}\right) - \left(-2e^{-2} - e^{-2}\right)\right) = 21e^{-6} - 9e^{-2}$$

Second term (using substitution $u = t - 2$, $t = u + 2$):

$$3\int_2^6 te^{-(t-2)}dt = 3\int_0^4 (u+2)e^{-u}du$$
$$= 3\left(\int_0^4 ue^{-u}du + 2\int_0^4 e^{-u}du\right)$$

Compute each integral:

First integral:

$$\int_0^4 ue^{-u}du = -ue^{-u}\Big|_0^4 - \int_0^4 -e^{-u}du = \left(-4e^{-4} + e^{-4} - 1\right) = -5e^{-4} + 1$$

Second integral:

$$2\int_0^4 e^{-u}du = 2\left(-e^{-u}\Big|_0^4\right) = 2(-e^{-4} + 1) = 2(1 - e^{-4})$$

Combine:

$$3\left((-5e^{-4} + 1) + 2(1 - e^{-4})\right) = 3\left(-7e^{-4} + 3\right) = -21e^{-4} + 9$$

Combine terms:

$$m_{1_2} = (21e^{-6} - 9e^{-2}) + (-21e^{-4} + 9) = 21e^{-6} - 9e^{-2} - 21e^{-4} + 9$$

Total m_1:

$$\begin{aligned} m_1 &= m_{1_1} + m_{1_2} \\ &= (9e^{-2} + 3) + (21e^{-6} - 9e^{-2} - 21e^{-4} + 9) \\ &= (9e^{-2} - 9e^{-2}) + (3 + 9) + (21e^{-6} - 21e^{-4}) \\ &= 12 + 21e^{-6} - 21e^{-4} \end{aligned}$$

Numerical value:

$$m_1 \approx 12 + 21(0.00247875) - 21(0.0183156) = 12 + 0.05205375 - 0.3846276 \approx 11.6674$$

Step 3: Calculate Delay Time t_d

$$t_d = \frac{m_1}{m_0} \approx \frac{11.6674}{5.9525} \approx 1.9601$$

Conclusion

- Total Area (m_0): Approximately 5.9525
- First Moment (m_1): Approximately 11.6674
- Delay Time (t_d): Approximately 1.9601

Aufgabe 3

a) die Ströme I_1, I_2 und I_3.

$R_{1|2} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$

$R_{1|2} = \frac{1}{\frac{1}{20\,k\Omega} + \frac{1}{30\,k\Omega}}$

$R_{1|2} \approx 12000\,\Omega = 12\,k\Omega$

$R_{total} = R_{1|2} + R_3 = 12\,k\Omega + 15\,k\Omega = 27\,k\Omega$

$I_q = \frac{U_q}{R_{total} + R_q} = \frac{10V}{27000\,\Omega + 2\,\Omega} = \frac{10V}{27002\,\Omega} \approx 0.00037\,A = 0.37\,mA$

$U_{R_3} = I_q \cdot R_3 = 0.37\,mA \cdot 15\,k\Omega = 0.37\,mA \cdot 15000\,\Omega = 5.55V$

$U_{1|2} = U_q - U_{R_3} = 10V - 5.55V = 4.45V$

* $I_1 = \frac{U_{1|2}}{R_1} = \frac{4.45V}{20\,k\Omega} = \frac{4.45V}{20000\,\Omega} = 0.0002225\,A = 0.2225\,mA$
* $I_2 = \frac{U_{1|2}}{R_2} = \frac{4.45V}{30\,k\Omega} = \frac{4.45V}{30000\,\Omega} = 0.0001483\,A = 0.1483\,mA$
* $I_3 = I_q = 0.37\,mA$

b) Bestimme den Spannungsabfall über dem Widerstand R_3.

$U_{R_3} = 5.55V$

c) die Ströme I_1', I_2' und I_3'.

$R_{3|mv} = \frac{1}{\frac{1}{R_3} + \frac{1}{R_{m}v}}$

$R_{3|mv} = \frac{1}{\frac{1}{15\,k\Omega} + \frac{1}{200\,k\Omega}}$

$R_{3|mv} =\approx 13953.48\,\Omega = 14\,k\Omega$

$R_{total\prime} = R_q + R_{mc} + R_{12} + R_{3|mv}$

$R_{total\prime} = 2\,\Omega + 8\,\Omega + 12000\,\Omega + 13946.73\,\Omega = 25956.73\,\Omega \approx 25.96\,k\Omega$

$I_{q\prime} = \frac{U_q}{R_{total\prime}} = \frac{10V}{26010\,\Omega} \approx 0.000385\,A = 0.385\,mA$

$U_{12\prime} = U_q - I_{q\prime} \cdot (R_q + R_{mc} + R_{3|mv})$

$U_{12\prime} = 10V - 0.385\,mA \cdot (2\,\Omega + 8\,\Omega + 13946.73\,\Omega) = 10V - 0.385\,mA \cdot 13956.73\,\Omega$

$U_{12\prime} = 10V - (0.385\,mA \cdot 13.95673\,k\Omega) = 10V - 5.38V = 4.62V$

* $I_1\prime = \frac{U_{12}\prime}{R_1} = \frac{4.62V}{20000\,\Omega} = 0.000231\,A = 0.231\,mA$
* $I_2\prime = \frac{U_{12}\prime}{R_2} = \frac{4.62V}{30000\,\Omega} = 0.000154\,A = 0.154\,mA$

$U_{3|mv}\prime = U_q - I_q\prime \cdot (R_q + R_{mc} + R_{1|2})$

$U_{3|mv}\prime = 10V - 0.385\,mA \cdot (2\,\Omega + 8\,\Omega + 12\,k\Omega) = 10V - 0.385\,mA \cdot 12010\,\Omega$

$U_{3|mv}\prime = 10V - (0.385\,mA \cdot 12.01\,k\Omega) = 10V - 4.625V = 5.375V$

$I_3\prime = \frac{U_{3|mv}\prime}{R_3} = \frac{5.375V}{15000\,\Omega} = 0.0003583\,A = 0.358\,mA$

d) Bestimme den absoluten Fehler zu den Strömen aus Aufgabenteil a).

$\Delta I_1 = |I_1\prime - I_1| = |0.231\,mA - 0.2225\,mA| = 0.0085\,mA$

$\Delta I_2 = |I_2\prime - I_2| = |0.154\,mA - 0.1483\,mA| = 0.0057\,mA$

$\Delta I_3 = |I_3\prime - I_3| = |0.358\,mA - 0.37\,mA| = 0.012\,mA$